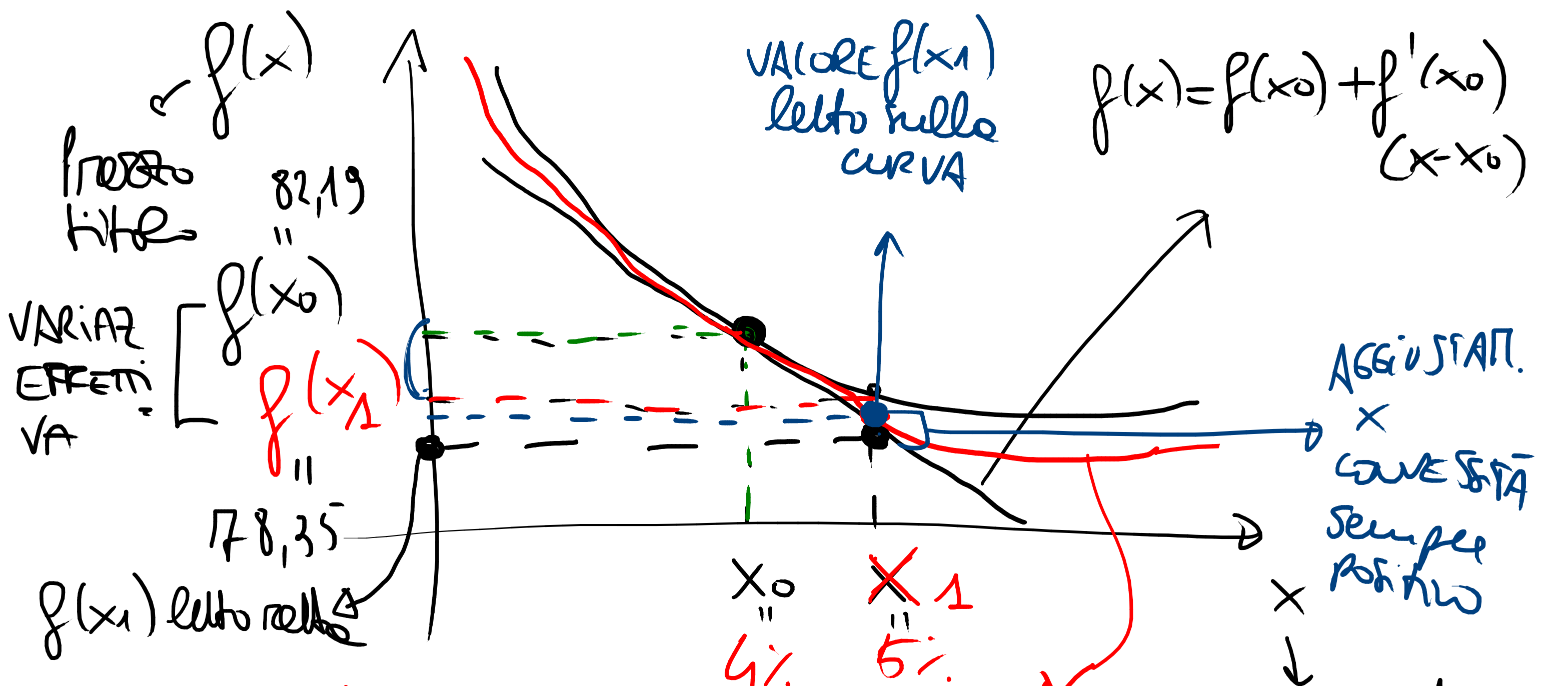
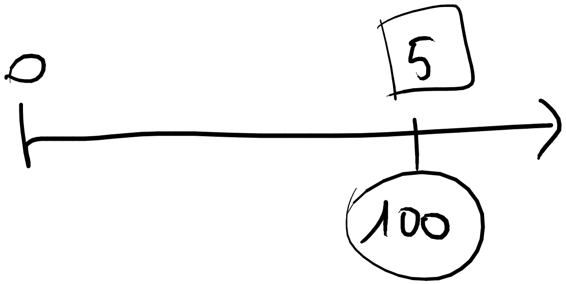
$$\underbrace{f(x) - f(x_0)}_{\Delta P} = f'(x_0)\underbrace{(x - x_0)}_{\Delta r} + \frac{1}{2} f''(x_0)\underbrace{(x - x_0)^2}_{\Delta r^2}$$



$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \underbrace{\frac{1}{2} f''(x_0)(x - x_0)^2}$$ Δ



$P(4\%) = 82{,}19$

$P(5\%) = 78{,}35$

$\Delta P =$ DIFFERENZA REALE $= 78{,}35 - 82{,}19 = -3{,}84$

$$\Delta P = -\frac{D}{1+r} \cdot P \cdot \Delta r \quad 1^\circ \text{ ORDINE}$$

$$\Delta P = -\boxed{\frac{5}{1+4\%}} \cdot 82{,}19\,(\underbrace{5\% - 4\%}_{1\%}) = -3{,}95$$

VARIAZIONE DI PREZZO STIMATA CON POLINOMIO TAYLOR 1° ORDINE

$$\frac{\Delta P}{82{,}19} = -\underbrace{DM}_{4{,}80} \cdot \underbrace{\Delta r}_{1\%} = -4{,}80\%$$

$$\Delta P = \overset{f'(x_0)}{\boxed{-\frac{D}{1+r}\cdot P}}\,\Delta r + \underbrace{\frac{1}{2}\,\overset{f''(x_0)}{\boxed{\frac{C}{(1+r)^2}\,P}}\,\Delta r^2}_{\text{AGGIUSTAMENTO x LA CONVESSITÀ}}$$

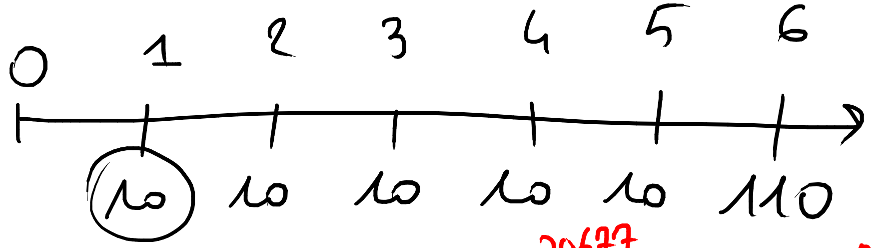
$$\Delta P = -3{,}95 + \underbrace{\frac{1}{2}\,\frac{5\cdot 6}{(1+4\%)^2}\cdot 82{,}19\cdot(0{,}01)^2}_{\text{AGGIUSTAM x CONVESSITÀ}} =$$

$$\Delta P = -3{,}95 + 0{,}114 = \underbrace{-3{,}836}$$

VARIAZIONE ottenuta con polinomio Taylor 2° ORDINE

EX COUPON BOND:

$r = 2\%$



$$\text{DURATION} = 1 \cdot \boxed{\frac{\frac{10}{1+2\%}}{144,81}}_{=p_1=0,0677} + 2 \cdot \boxed{\frac{\frac{10}{(1+2\%)^2}}{144,81}}_{=p_2=0,066} + 3 \cdot \boxed{\frac{\frac{10}{(1+2\%)^3}}{144,81}}_{=p_3=0,065}$$

$$+ 4 \cdot \boxed{\frac{\frac{10}{(1+2\%)^4}}{144,81}}_{=p_4=0,0637} + 5 \cdot \boxed{\frac{\frac{10}{(1+2\%)^5}}{144,81}}_{=p_5=0,0625} + 6 \cdot \boxed{\frac{\frac{110}{(1+2\%)^6}}{144,81}}_{=p_6=0,6745} =$$

$$\text{DURATION} \rightarrow 5,0062$$

$$\sum_{k=1}^{6} p_k = 1$$

$$\text{Prezzo} = \frac{10}{1+2\%} + \frac{10}{(1+2\%)^2} + \frac{10}{(1+2\%)^3} + \frac{10}{(1+2\%)^4} + \frac{10}{(1+2\%)^5} + \frac{110}{(1+2\%)^6}$$

Prezzo = 144,81

CONVEXITY : $\sum_{k=1}^{n} t_k (t_k+1) \cdot p_k$

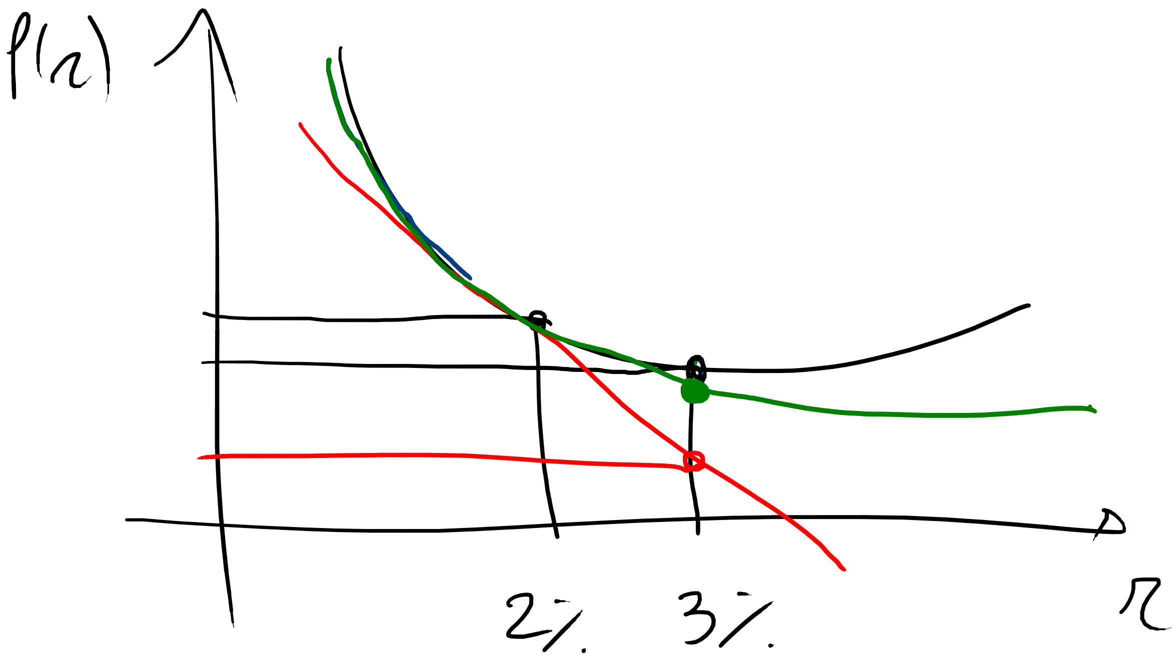
DURATION: $\sum_{k=1}^{n} t_k \cdot p_k$

$$p_k = \frac{\dfrac{C_k}{(1+r)^{t_k}}}{\text{PREZZO}}$$

$$\frac{100}{(1+4\%)^5} = 82,19$$

$$\text{DURAT} = \frac{5 \cdot \dfrac{100}{(1+4\%)^5}}{82,19} = 5 \cdot 1 = 5$$

CONVEXITY $= (5)(5+1) \cdot 1$



P(r)
2%
3%
r

$$G = 1 \cdot 2 \cdot p_1 + 2 \cdot 3 \cdot p_2 + 3 \cdot 4 \cdot p_3 + 4 \cdot 5\, p_4 +$$
$$5 \cdot 6\, p_5 + 6 \cdot 7 \cdot p_6$$

$$G = k(k+1) \cdot p_k =$$

$$G = 2 \cdot 0{,}0677 + 6 \cdot 0{,}066 + 12 \cdot 0{,}065 + 20 \cdot 0{,}0637$$
$$+ 30 \cdot 0{,}0625 + 42 \cdot 0{,}6745 = 32{,}77$$

SE TASSO PASSA 2% AL 3%

$$\Delta P \text{ reale} = P(3\%) - P(2\%) =$$

$$\Delta P \text{ reale} = 137{,}92 - 144{,}81 = -6{,}89\ldots$$

$$P(3\%) = \frac{10}{(1+3\%)} + \frac{10}{(1+3\%)^2} + \frac{10}{(1+3\%)^3} + \frac{10}{(1+3\%)^4} + \frac{10}{(1+3\%)^5} + \frac{110}{(1+3\%)^6}$$

$$= 10 \cdot a_{\overline{6}|3\%} + \frac{100}{(1+3\%)^6} = 137{,}92$$

$$\Delta P = -\frac{D}{1+r} \cdot P \cdot \Delta r = -\frac{5{,}0062}{(1+2\%)} \cdot 144{,}81 \cdot 1\% = \boxed{-7{,}11}$$

1° ORDINE

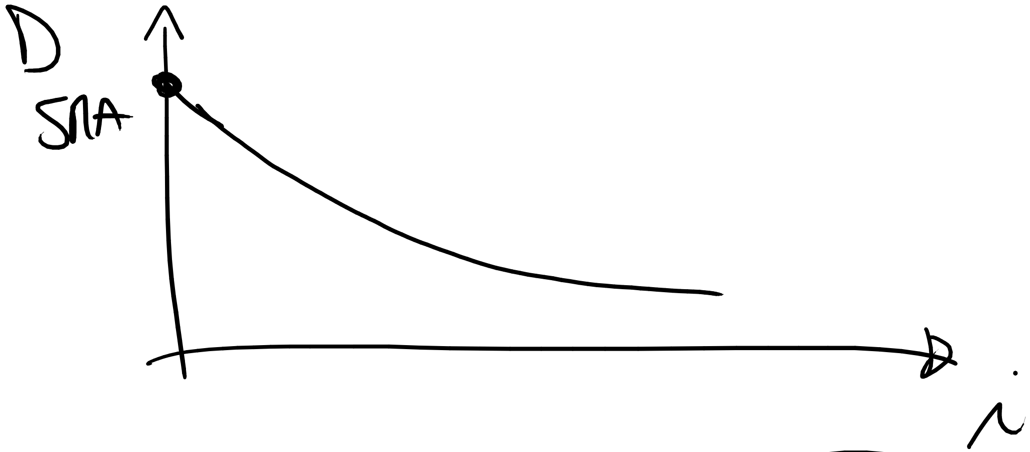
$$\Delta P = -7{,}11 + \frac{1}{2}\frac{C}{(1+r)^2} \cdot P \cdot \Delta r^2 =$$

$$\Delta P = -7{,}11 + \underbrace{\frac{1}{2}\frac{32{,}77}{(1+2\%)^2} \cdot 144{,}81 \cdot (1\%)^2}_{} =$$

$$\Delta P = -7{,}11 + 0{,}22 = \boxed{-6{,}89\ldots}\ldots$$

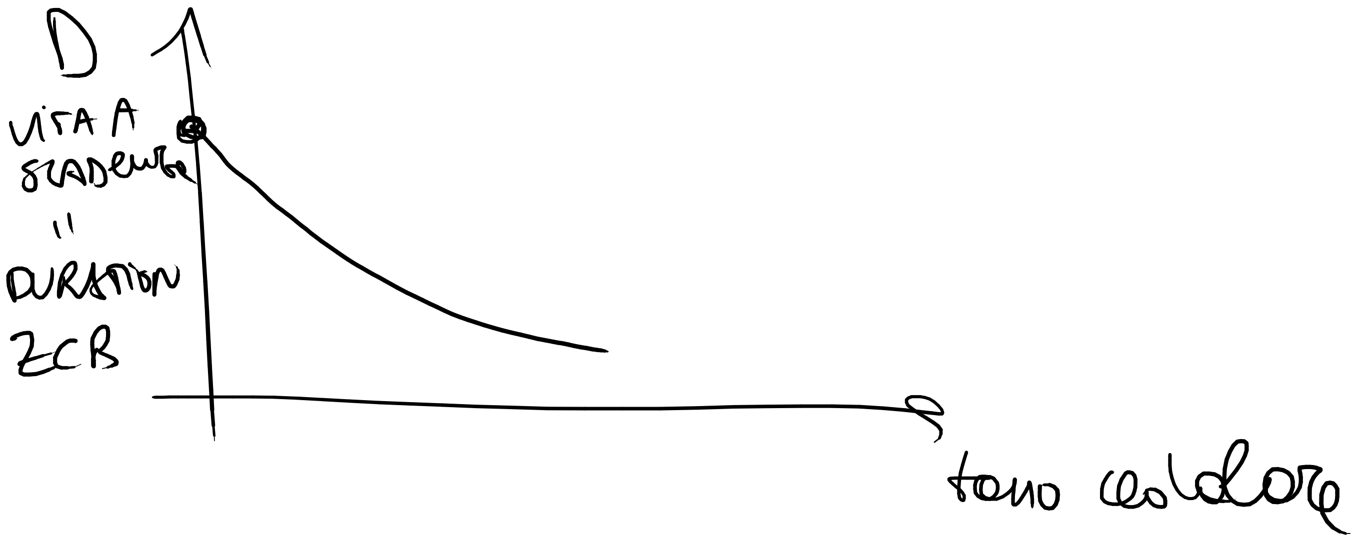
2° ORDINE

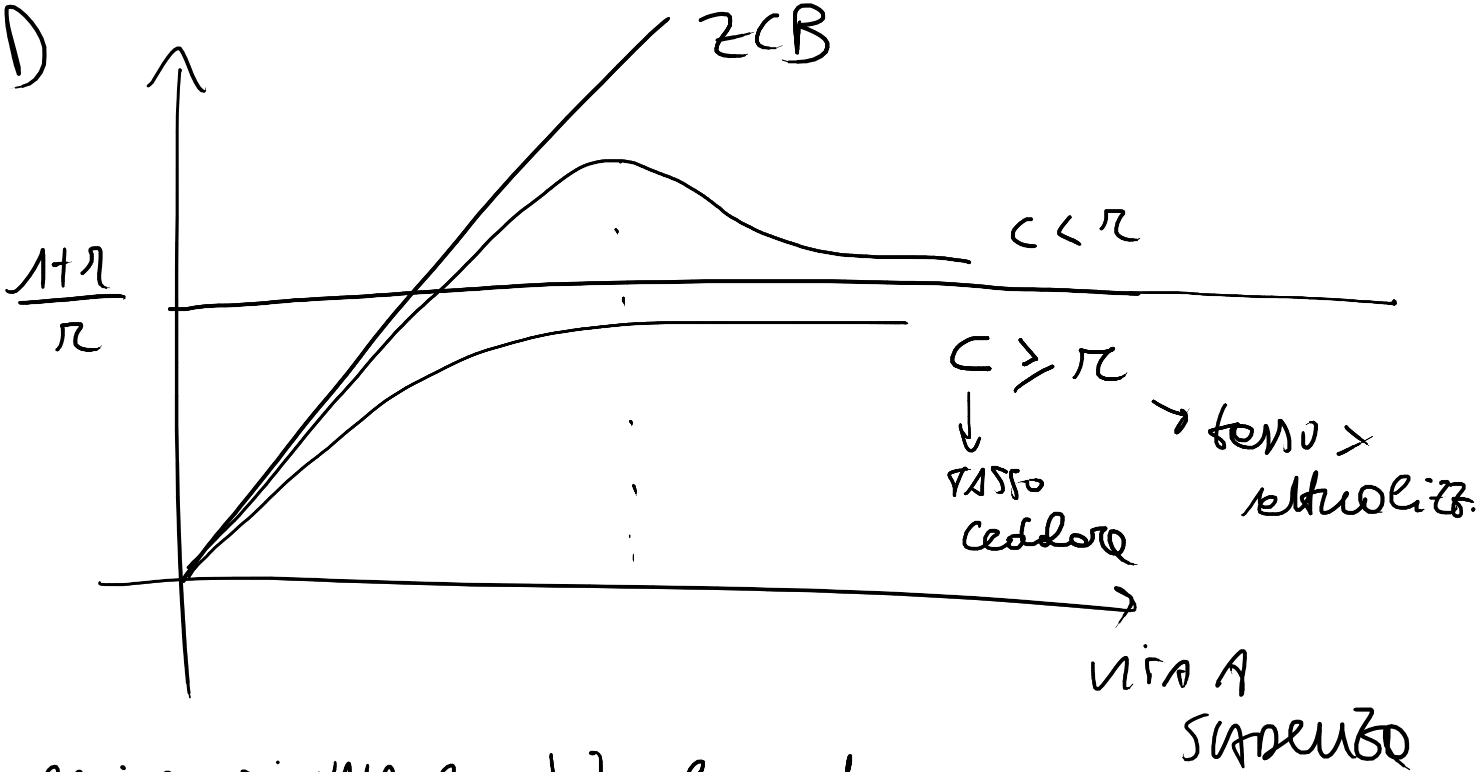
PROPRIETÀ DELLA DURATION:



$$D = \sum t_k \cdot \frac{c_k(1+i)^{-k}}{\text{Prezzo}}$$

$$i=0 \quad D = \sum t_k \cdot \frac{c_k}{\sum c_k} = SMA$$


D
VITA A SCADENZA
=
DURATION
ZCB
tasso cedolare



DURATION DI UNA Rendita Perpetua:

$$P(r) = \boxed{\frac{C}{r}} \qquad P'(r) = -\frac{C}{r^2} = -\frac{D}{1+r} \cdot P \qquad D = \frac{1+r}{r}$$

$$\cancel{-}\frac{\cancel{P}}{r} = \cancel{-}\frac{D}{1+r} \cdot \cancel{P} \rightarrow$$