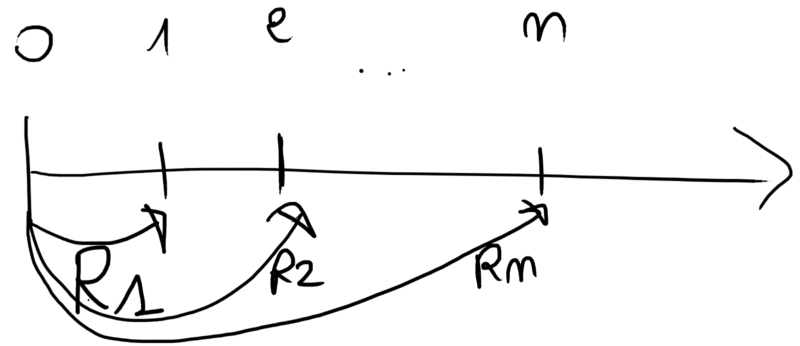
STRUTTURA PER SCADENZA DEI TASSI DI INTERESSE:

A PRONTI

A TERMINE

→ STRUTTURA A PRONTI = TASSI DI INTERESSE A PRONTI

TASSI SPOT

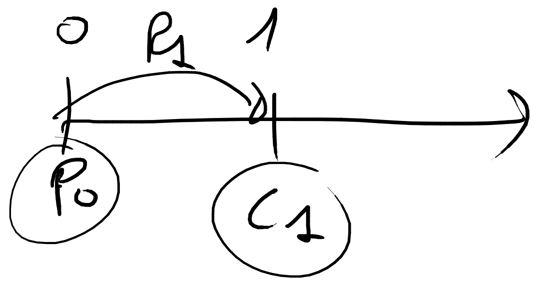


$R_1, R_2, \ldots, R_n$ li otteniamo DA

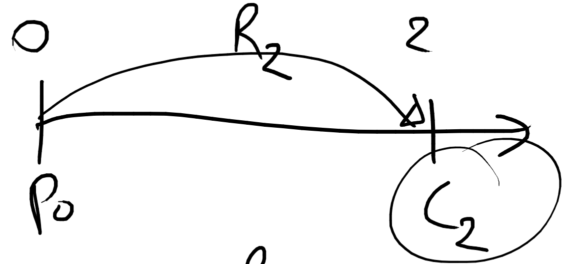
TITOLI OBBLIGAZIONARI senza cedole che scadono

TRA 1, 2, n periodi

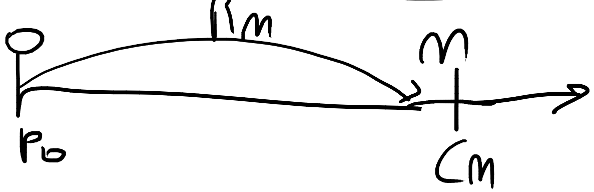
$$P_0 = \frac{C_1}{(1+R_1)^1}$$



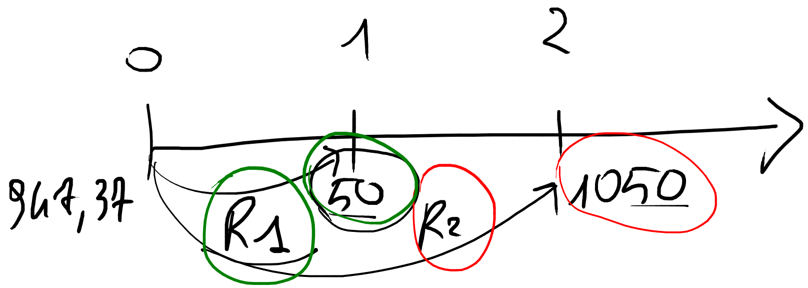
$$P_0 = \frac{C_2}{(1+R_2)^2}$$



$$P_0 = \frac{C_n}{(1+R_n)^n}$$



Sul mercato non ∃ titoli senza cedole con scadenze lunghe → obbligato estrarre i tassi a pronti dai titoli con cedole:



COUPON BOND = $947{,}37 = P_0 =$

$R_1 = 0{,}06$

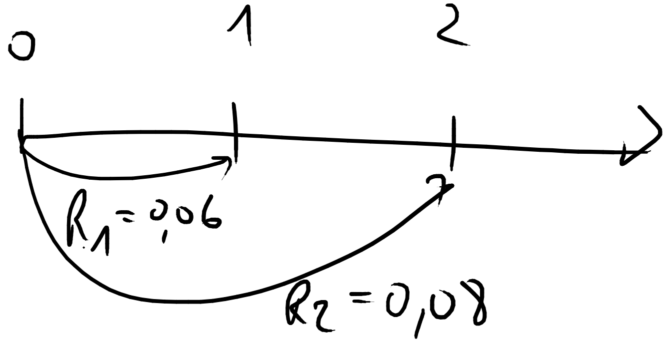
$$\boxed{947{,}37 = \frac{50}{(1+0{,}06)^1} + \frac{1050}{(1+R_2)^2}}$$

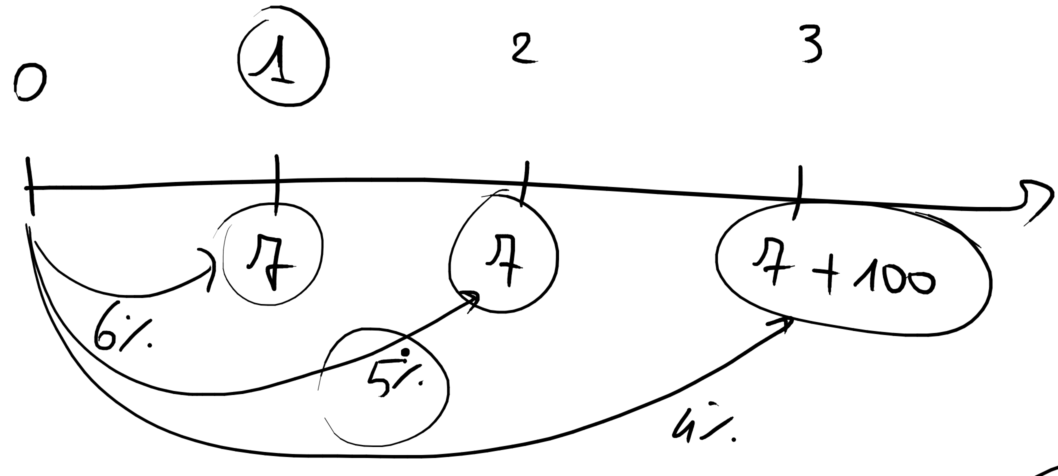
$$900{,}20 = \frac{1050}{(1+R_2)^2}$$

$$(1+R_2)^2 = \frac{1050}{900{,}20}$$

$$R_2 = \sqrt[2]{\frac{1050}{900{,}20}} - 1$$

$$R_2 = 0{,}08$$





$$P = \frac{7}{(1+6\%)^1} + \frac{7}{(1+5\%)^2} + \frac{107}{(1+4\%)^3} = 108,07$$

NON ARBITRAGGIO $\quad P = 108,07$

	0	1	2	3
COMPRO 7 UNITÀ ZCB SCADE IN 1	$-\frac{7}{1+6\%}$	7		
COMPRO 7 UNITÀ ZCB SCADE 2	$-\frac{7}{(1+5\%)^2}$		7	
COMPRO 107 UNITÀ ZCB SCADE IN 3	$-\frac{107}{(1+4\%)^3}$			107
VENDO COUPON BOND	P	-7	-7	-107
$P-\frac{7}{1+6\%}-\frac{7}{(1+5\%)^2}-\frac{107}{(1+4\%)^3}=0$				

se $P > 108{,}07$ VENDO COUPON BOND
COMPRO IL PORTAFOGLIO
DI ZCB

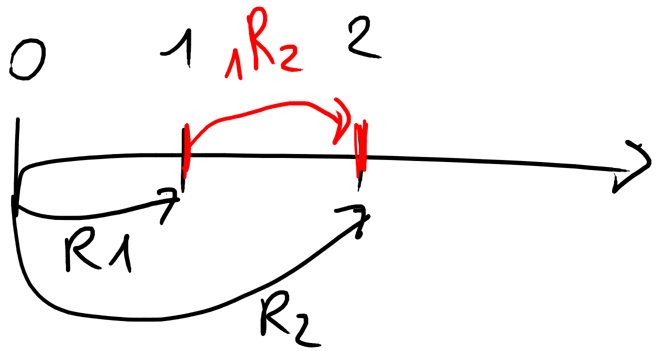
se $P < 108{,}07$ COMPRO CB
VENDO GLI ZCB

QUESTI SAREBBERO ARBITRAGGI

QUINDI PREZZO CB = SOMMA PREZZI ZCB

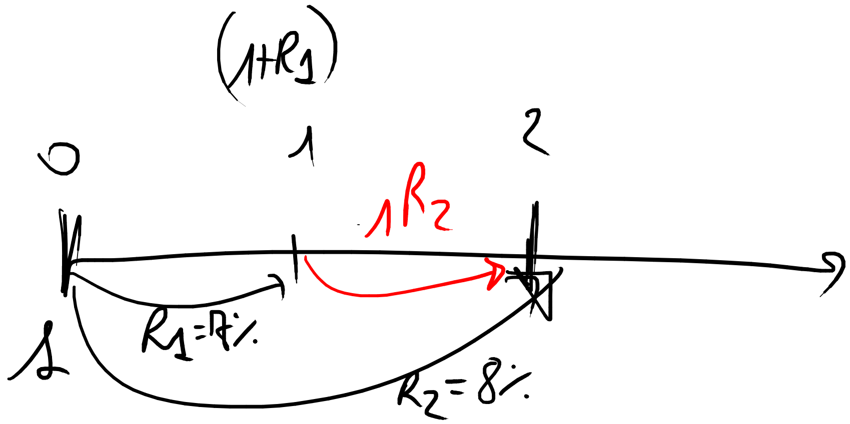
$$108{,}07 = P = \frac{7}{(1+6\%)} + \frac{7}{(1+5\%)^2} + \frac{107}{(1+4\%)^3}$$

TASSI FORWARD = STRUTTURA X SCADENZA
A TERMINE A TERMINE



$_1R_2$ serve x valorizzare un contratto differito nel tempo

TASSI FORWARD sono implicititi nei TASSI A PRONTI



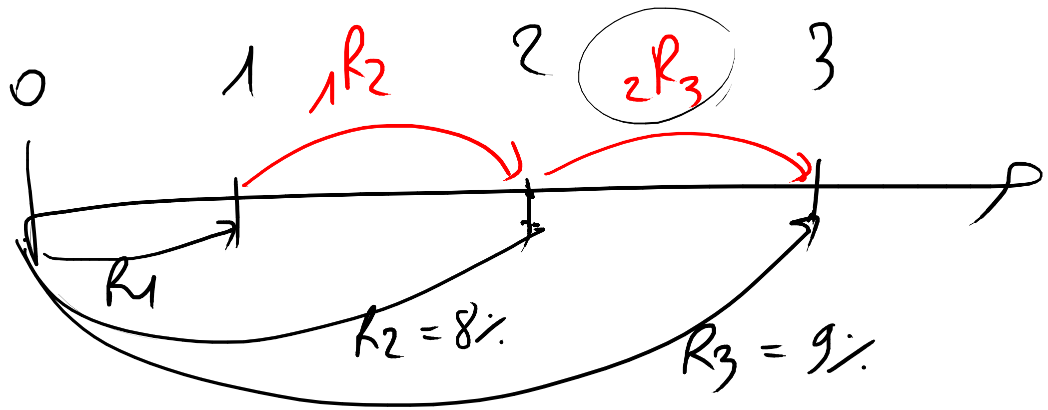
$$(1+R_2)^2 = (1+R_1)^1 (1+{}_1R_2)^1$$

8%　　　　7%

$$(1+{}_1R_2) = \frac{(1+8\%)^2}{1+7\%}$$

$${}_1R_2 = \frac{(1+8\%)^2}{1+7\%} - 1$$

$$= 9{,}01\%$$



$R_2 = 8\%$

$R_3 = 9\%$

$_2R_3 =$

$$(1+R_3)^3 = (1+R_2)^2 (1+{}_2R_3)$$

$$(1+9\%)^3 = (1+8\%)^2 (1+{}_2R_3)$$

$${}_2R_3 = \frac{(1+9\%)^3}{(1+8\%)^2} - 1$$

$${}_2R_3 = 11{,}03\%$$

EX:

	0	1	2	3
CB	98	9	9	109
CB	96	7	107	
ZCB	92	100		

$_1R_2 = 10{,}02\%$

$_2R_3 =$

$R_1 = 8{,}6\%$

$R_2 = 9{,}31\%$

$R_3 = 9{,}87\%$

$$92 = \frac{100}{1+R_1} \qquad R_1 = \frac{100}{92} - 1 = 0{,}086$$

$$96 = \frac{7}{(1+8{,}6\%)} + \frac{107}{(1+R_2)^2} \qquad (1+R_2)^2 = \frac{107}{89{,}55}$$

$$R_2 = \sqrt{\frac{107}{89{,}55}} - 1 = 9{,}31\%$$

$$98 = \frac{9}{(1+8{,}6\%)} + \frac{9}{(1+9{,}31\%)^2} + \frac{109}{(1+R_3)^3}$$

$$(1+R_3)^3 = \frac{109}{82{,}18}$$

$$R_3 = \sqrt[3]{\frac{109}{82{,}18}} - 1$$

$$R_3 = 0{,}0987$$

$$(1+R_2)^2 = (1+R_1)(1+{}_1R_2)$$

$$(1+9{,}31\%)^2 = (1+8{,}6\%)(1+{}_1R_2)$$

$${}_1R_2 = \frac{(1+9{,}31\%)^2}{1+8{,}6\%} - 1$$

$${}_1R_2 = 10{,}02\%$$

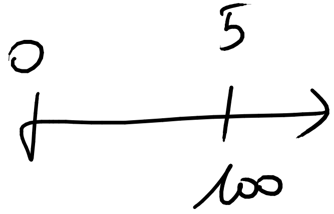
$$(1+R_3)^3 = (1+R_2)^2 (1+{}_2R_3)$$

$${}_2R_3 = \frac{(1+R_3)^3}{(1+R_2)^2} - 1$$

$${}_2R_3 = \frac{(1+9{,}87\%)^3}{(1+9{,}31\%)^2} - 1$$

$${}_2R_3 = 10{,}99\%$$

MISURA RISCHIO TASSO :



es: ZCB 5 ANNI $R_5 = 4\%$

$$P = \frac{100}{(1+4\%)^5} = 82,19$$

$R_5 = 5\%$

$$P = \frac{100}{(1+5\%)^5} = 78,35$$

$\Delta r = 1\%$

$$\boxed{\Delta P = -3,84}$$

$P(r)$

VEDIAMO COME APPROSSIMARE ΔP

$$P = \sum c_k (1+r)^{-k}$$

r COSTANTE $\forall$ SCADENZA

STRUTTURA X SCADENZA PIATTA

$\Delta P \quad P(r) = f(x)$

$$\overbrace{f(x) - f(x_0)}^{\Delta P} = f'(x_0) \overbrace{(x - x_0)}^{\Delta r} \qquad 1^{\circ} \text{ ORDINE}$$

$\cong -3{,}84 \qquad 5\% \quad 4\%$

$$\underbrace{f(x) - f(x_0)}_{\Delta P} = \underbrace{f'(x_0)(x - x_0)}_{} + \underbrace{\frac{1}{2} f''(x_0) \overbrace{(x - x_0)^2}^{\Delta r^2}}_{2^{\circ} \text{ ORDINE}}$$

AGGIUSTAMENTO X LA CONVESSITÀ

$$P = \sum c_k (1+r)^{-k}$$

$$P'(r) = \sum c_k (-k)(1+r)^{-k-1}$$

$$\boxed{P''(r) = \sum c_k (-k)(-k-1)(1+r)^{-k-2}}$$

$$\Delta P = \left[\sum c_k (-k)(1+r)^{-k-1}\right] \Delta r \qquad 1^\circ \text{ ordine}$$

$$= \frac{\sum c_k (-k)(1+r)^{-k}}{(1+r)} \cdot \frac{P}{P} \cdot \Delta r$$

$$-D = \frac{\sum k \cdot c_k (1+r)^{-k}}{P}$$

$$\Delta P = -\frac{D}{1+r} \cdot P \cdot \Delta r$$

$$\Delta P = -\boxed{\frac{D}{1+r}} \cdot P \cdot \Delta r$$

DURATION
MODIFICATA

$$\Delta P = -DM \cdot P \cdot \Delta r$$

$$\frac{\Delta P}{P} = -DM \cdot \Delta r$$

$$-5\% = -5 \cdot 1\%$$

DURATION E DURATION MODIFICATA SONO INDICATORI DI RISCHIO DI TASSO DEL TITOLO

$$P''(r) = \sum c_k(-k)(-k-1)\underbrace{(1+r)^{-k-2}}$$

$$= \frac{\sum c_k\, k(k+1)(1+r)^{-k}}{(1+r)^2} \cdot \frac{P}{P}$$

$$\sum k(k+1) \cdot p_k = \text{CONVEXITY}$$

$$\hookrightarrow \frac{c_k (1+r)^{-k}}{P}$$

$$\sum k \cdot p_k = \text{DURATION}$$

$$P''(r) = \frac{C \cdot P}{(1+r)^2}$$

$$\Delta P = -\frac{D}{(1+r)} \cdot P \, \Delta r + \frac{1}{2} \frac{C \cdot P}{(1+r)^2} \Delta r^2$$

$$P(x) - P(x_0) = P'(x_0)(x - x_0) + \frac{1}{2} P''(x_0) \cdot (x - x_0)^2$$