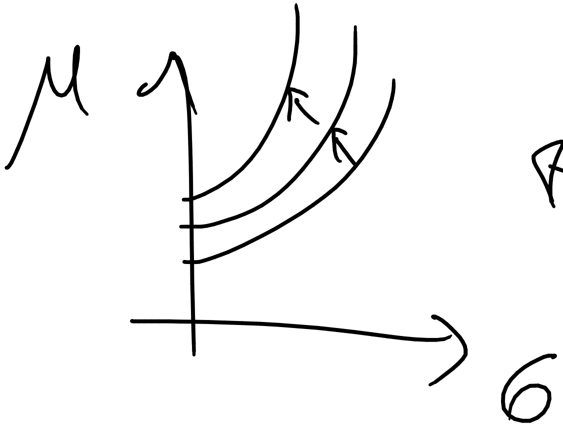
monday 18 h. 14.00

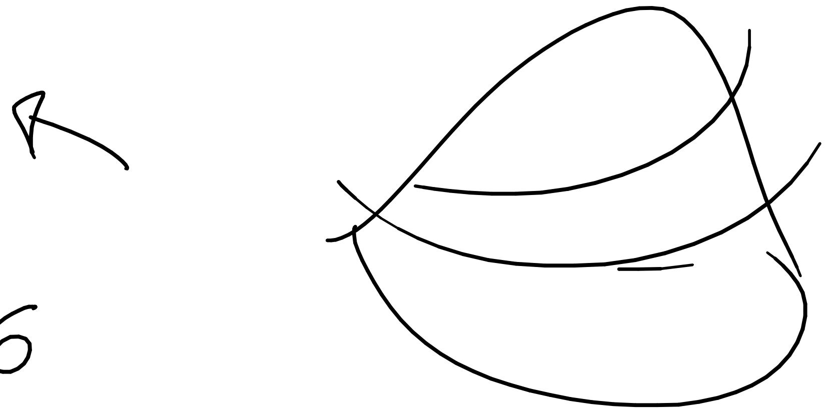
wednesday 20 8,30 – 12

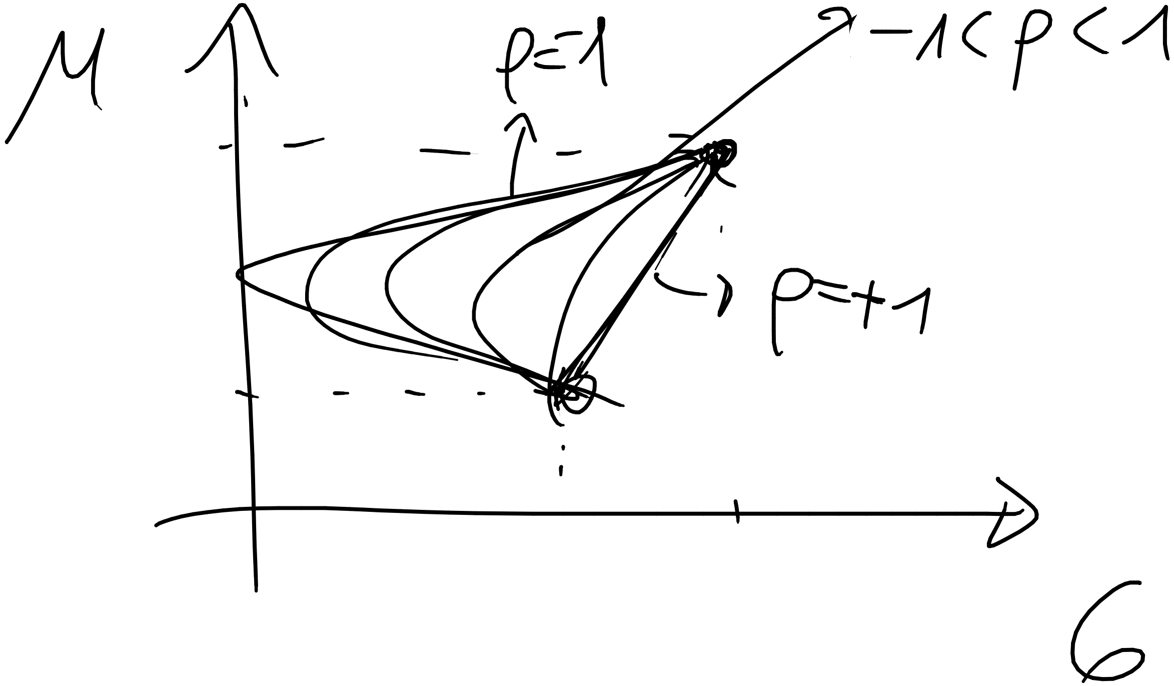
MARKOWITZ: 1) one period

2) only mean and variance

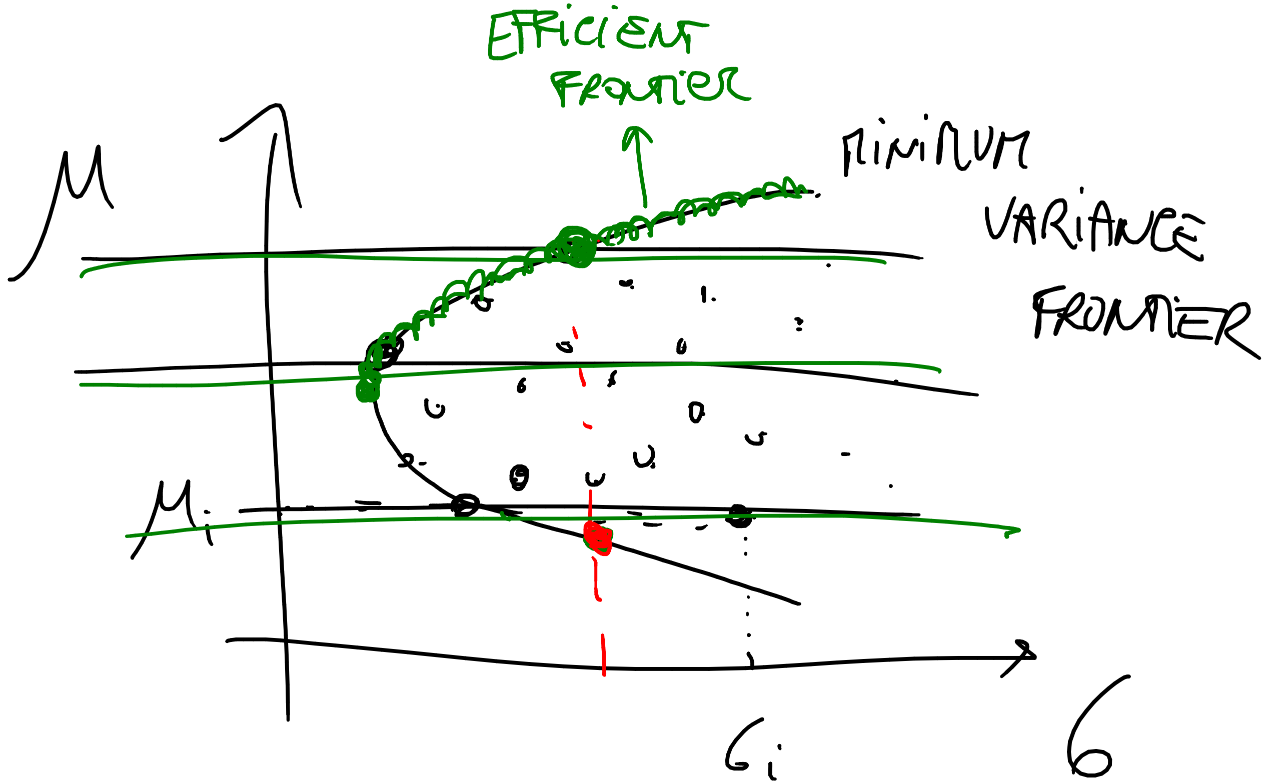
3) utility function (μ, σ^2)

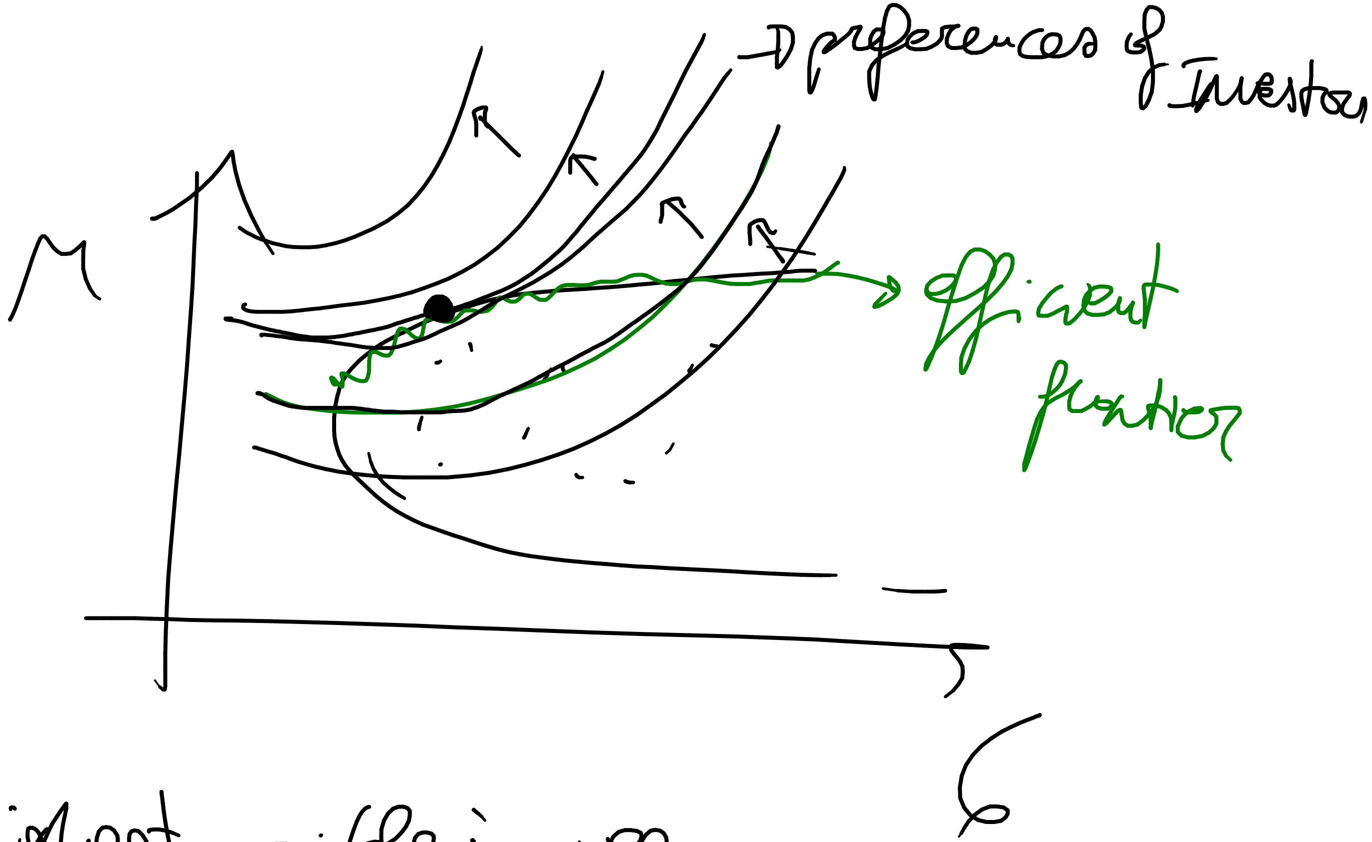






2 Assets

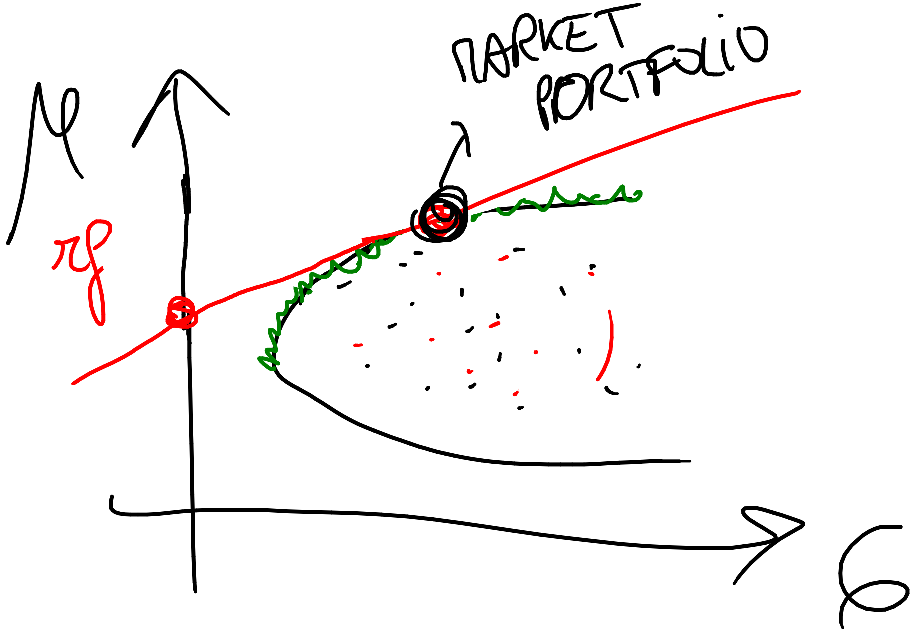

EFFICIENT FRONTIER
MINIMUM VARIANCE FRONTIER
μ
μ_i
σ_i
σ



highest possible indifference curve
tangent to the efficient frontier

CAPM = CAPITAL ASSET PRICING MODEL

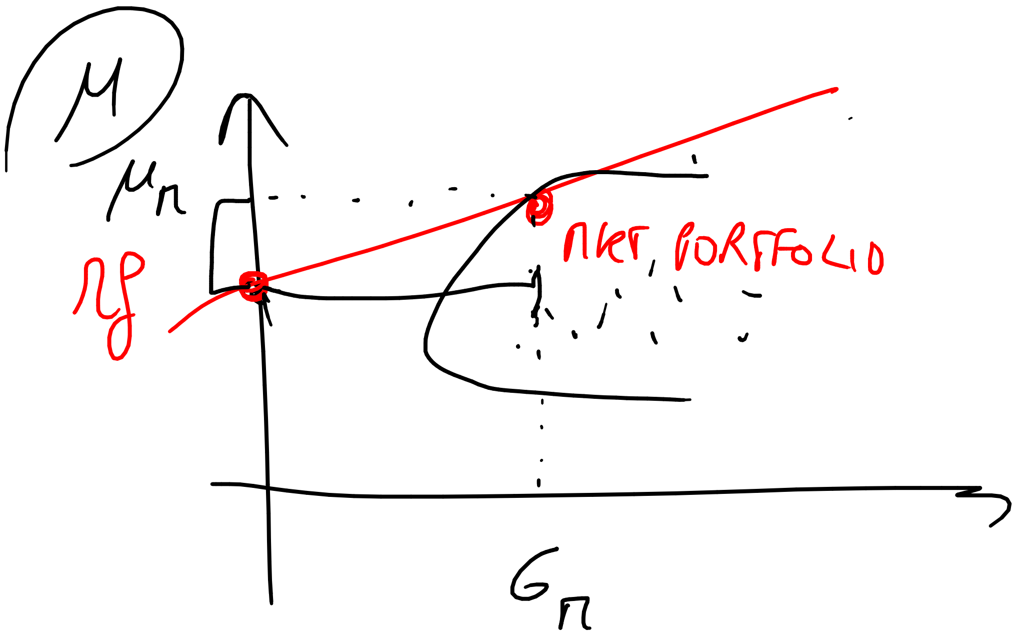
financial equilibrium :



1) μ, σ

2) demand = supply

3) ∃ risk free asset



CAPITAL
MARKET
LINE

$$E(R_i) = r_f + \left[\frac{E(R_M) - r_f}{\sigma_M}\right] \cdot \sigma_i$$

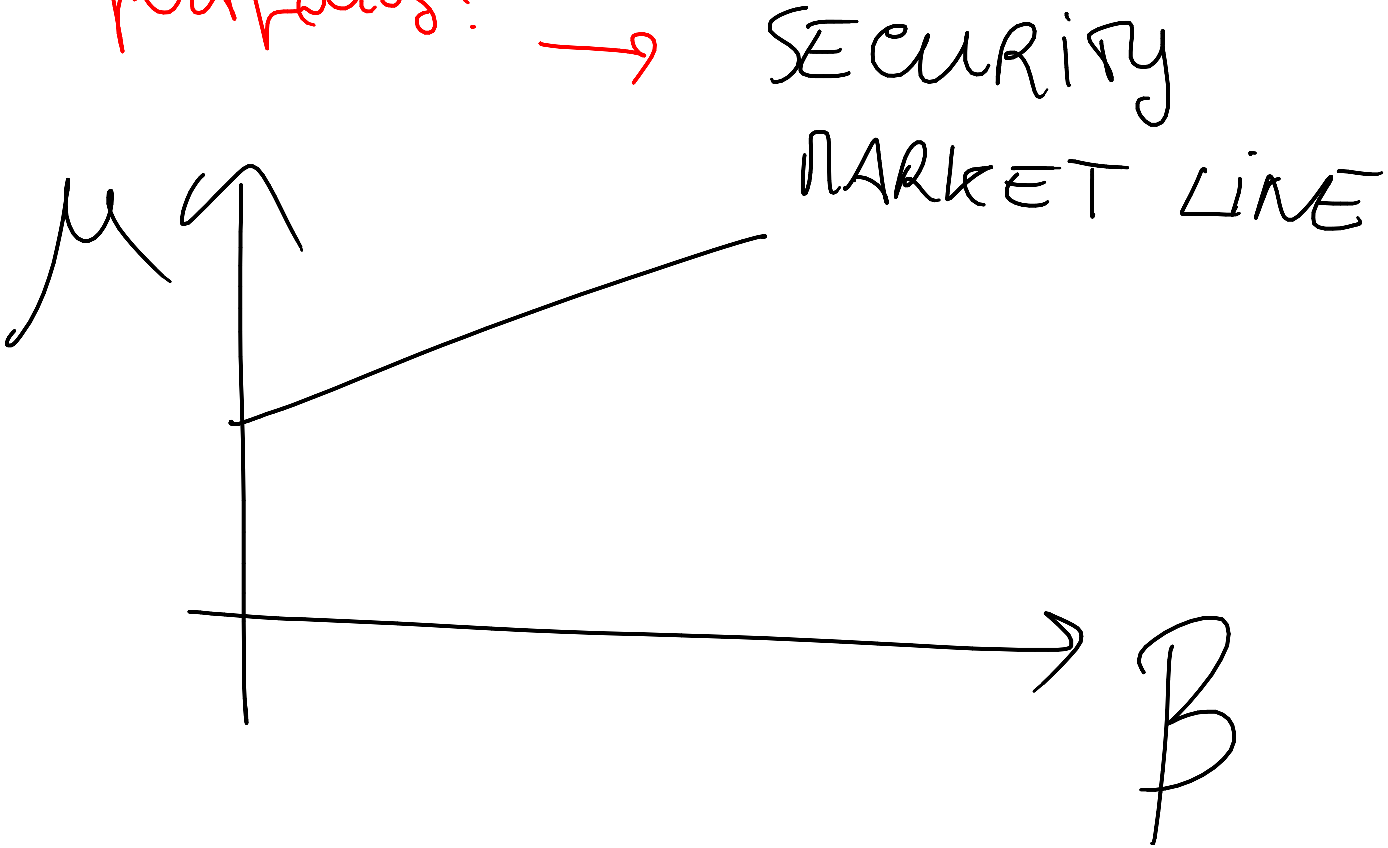
$$\parallel$$

$$\mu_i = r_f + \left[\frac{\mu_M - r_f}{\sigma_M}\right] \cdot \sigma_i$$

$$\underbrace{E(R_i) - r_f}_{\text{MARKET RISK PREMIUM}} = \underbrace{\frac{E(R_m) - r_f}{\sigma_m}}_{\text{PRICE OF RISK}} \cdot \underbrace{\sigma_i}_{\text{QUANTITY OF RISK}}$$

CML APPLIES ONLY TO EFFICIENT PORTFOLIOS

What can we say about other portfolios?



$$E(R_i) = r_f + [E(R_m) - r_f] \cdot \beta_i$$

$$E(R_i) = r_f + [E(R_m) - r_f] \cdot \frac{cov(R_i, R_m)}{\sigma_m^2}$$

$$cov(R_i, R_m) = \rho_{im} \cdot \sigma_i \cdot \sigma_m$$

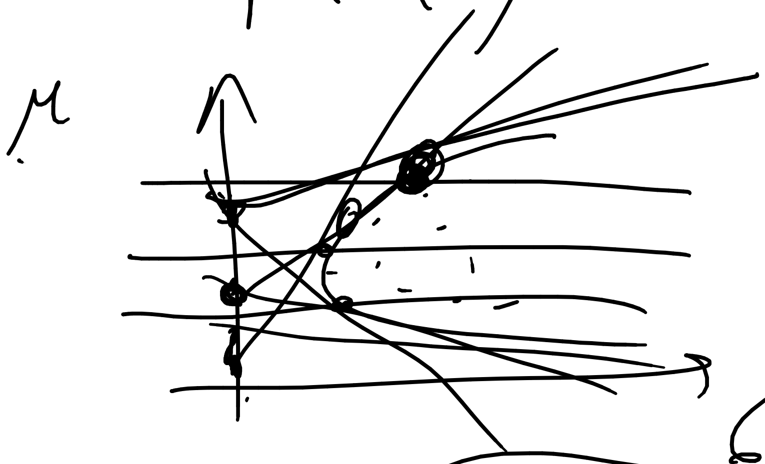
$$E(R_i) - r_f = \underbrace{[E(R_m) - r_f]}_{\text{PRICE OF RISK}} \cdot \frac{\rho_{im} \cdot \sigma_i \cdot \cancel{\sigma_m}}{\cancel{\sigma_m^2}}$$

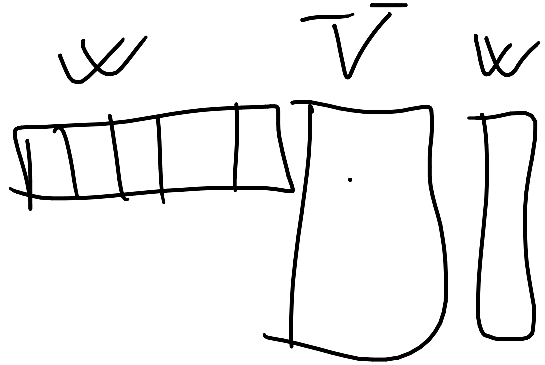
QUANTITY OF RISK

only the PART ρ_{im} is PRICED

MINIMUM VARIANCE FRONTIER:

$\forall$ fixed μ I want min σ^2





$$\begin{cases} \min \frac{1}{2} w^T \bar{V} w \\ \text{s.t.} \quad w^T \bar{R} + (1 - w^T) r_f = \mu \end{cases}$$

$$Z = V^{-1} \left(E(R) - C \right)$$

NON NORMALIZED PROPORTION

inverse VAR-COV MATRIX

mean returns

constant (risk free rate)

$$w_i = \frac{z_i}{\sum_i z_i}$$

USE TWO CONSTANTS

0

0,1

VAR-COV MATRIX

E(R)

E(R)-0

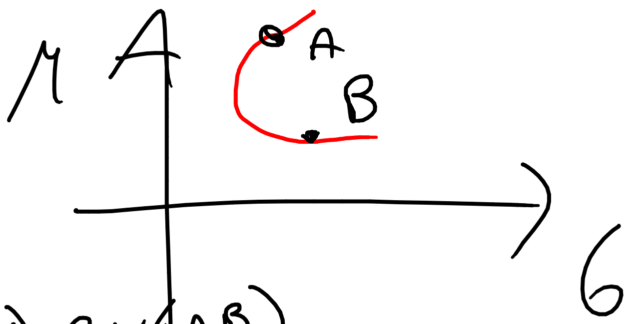
E(R)-0,1

z_2

$$z_1 = V^{-1} \cdot [E(R) - C]$$

= MMULT[MINVERSE(V); vector]

IF I HAVE 2 PORTFOLIOS WHICH ARE MINIMUM VARIANCE I CAN DERIVE ALL THE OTHERS ON THE MINIMUM VARIANCE FRONTIER BY A CONVEX COMBINATION OF THE TWO PORTFOLIOS



$$\mu_p = \alpha \cdot \mu_A + (1-\alpha) \cdot \mu_B$$

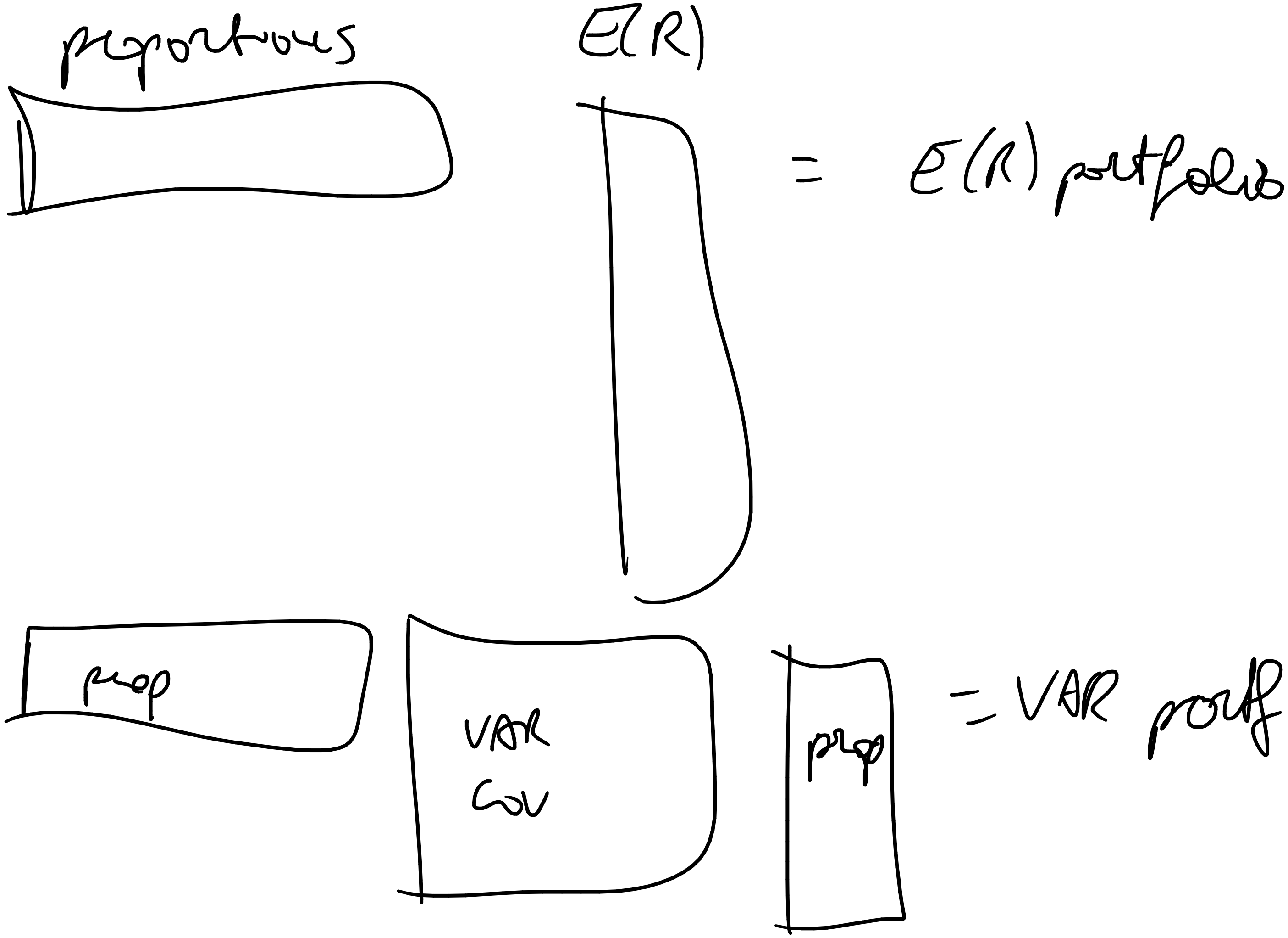
$$\sigma_p^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha) \cdot \mathrm{cov}(AB)$$

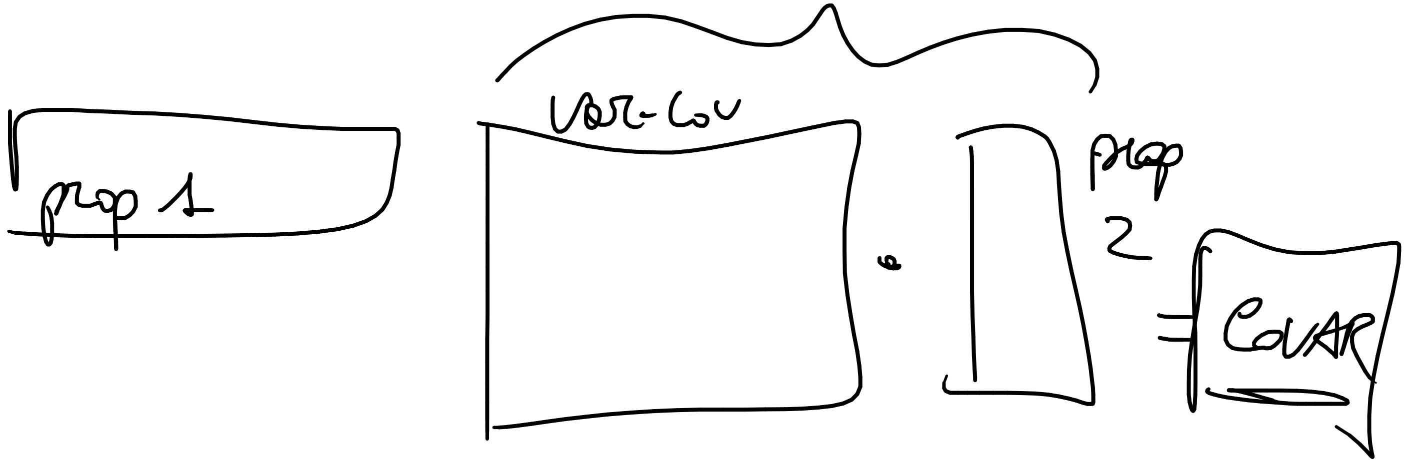
once I have $z_1 = V^{-1}[E(R) - 0{,}1]$

$$z_2 = V^{-1}[E(R)]$$

$$x_i = \frac{z_i^1}{\sum_i z_i^1}$$

$$y_i = \frac{z_i^2}{\sum_i z_i}$$


proportions
E(R)
= E(R) portfolio
prop
VAR
COV
prop
= VAR portf



MMULT[TRANSPOSE($prop_1$);
MMULT[var-cov; $prop_2$]]

GREEN PORTF $\mu_x \rightarrow \alpha$

ORANGE PORTF $\mu_y \rightarrow (1-\alpha)$

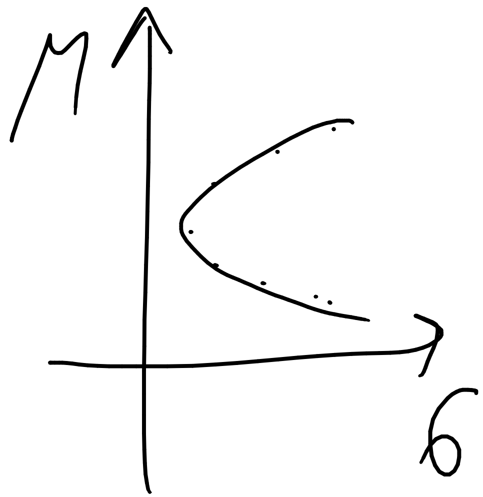
· $\mu_p = \alpha \cdot \mu_x + (1-\alpha) \cdot \mu_y$

· $VAR_p = \alpha^2 \cdot VAR\,x + (1-\alpha)^2 \cdot VAR\,y + 2 \cdot \alpha \cdot (1-\alpha) \cdot COV(x,y)$

$\downarrow$

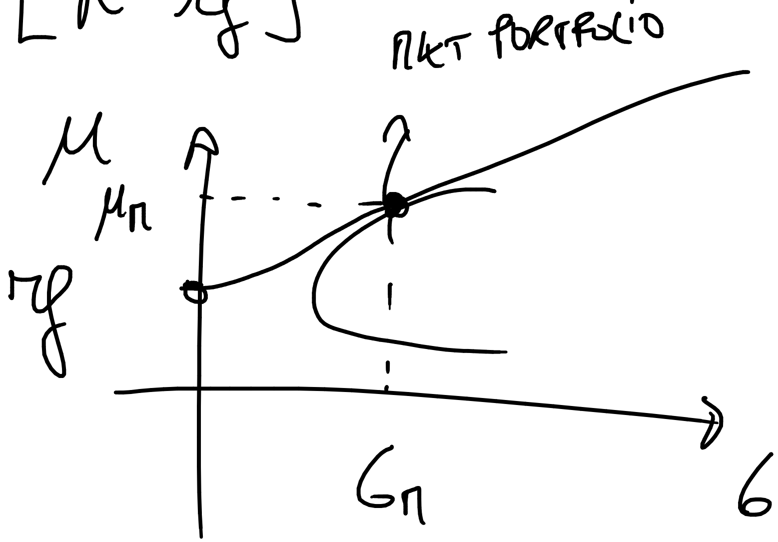
$STDDEV_p = \sqrt{VAR_p}$

α	$STDDEV_p$	$mean_p$



IN ORDER TO ESTIMATE THE MARKET PORTFOLIO: WE USE CONSTANT = RISK free rate

$$z = S^{-1}[R - r_f]$$



CAPITAL MARKET LINE :

μ_M

$$E(R_i) = r_f + \left[\frac{E(R_M) - r_f}{\sigma_M} \right] \cdot \sigma_i$$

CONSTANT

σ_M