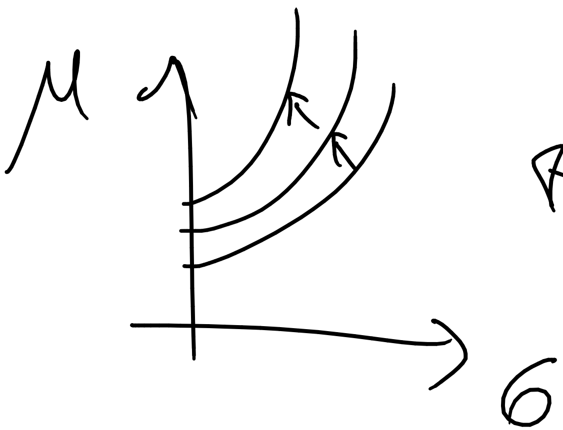
monday 18 h. 14.00

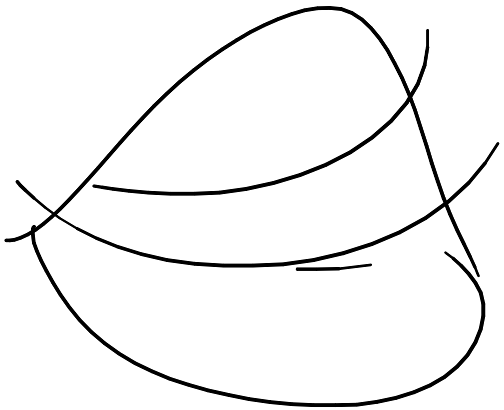
wednesday 20 8,30 - 12

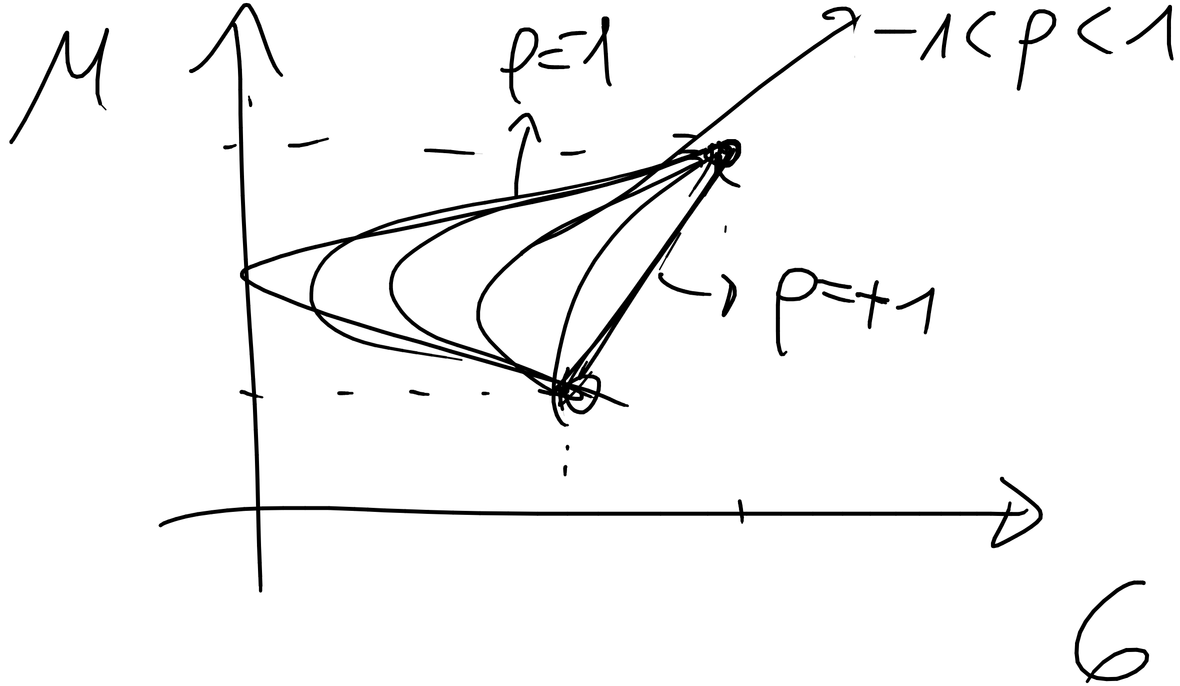
MARKOWITZ: 1) one PERIOD

2) only mean and variance

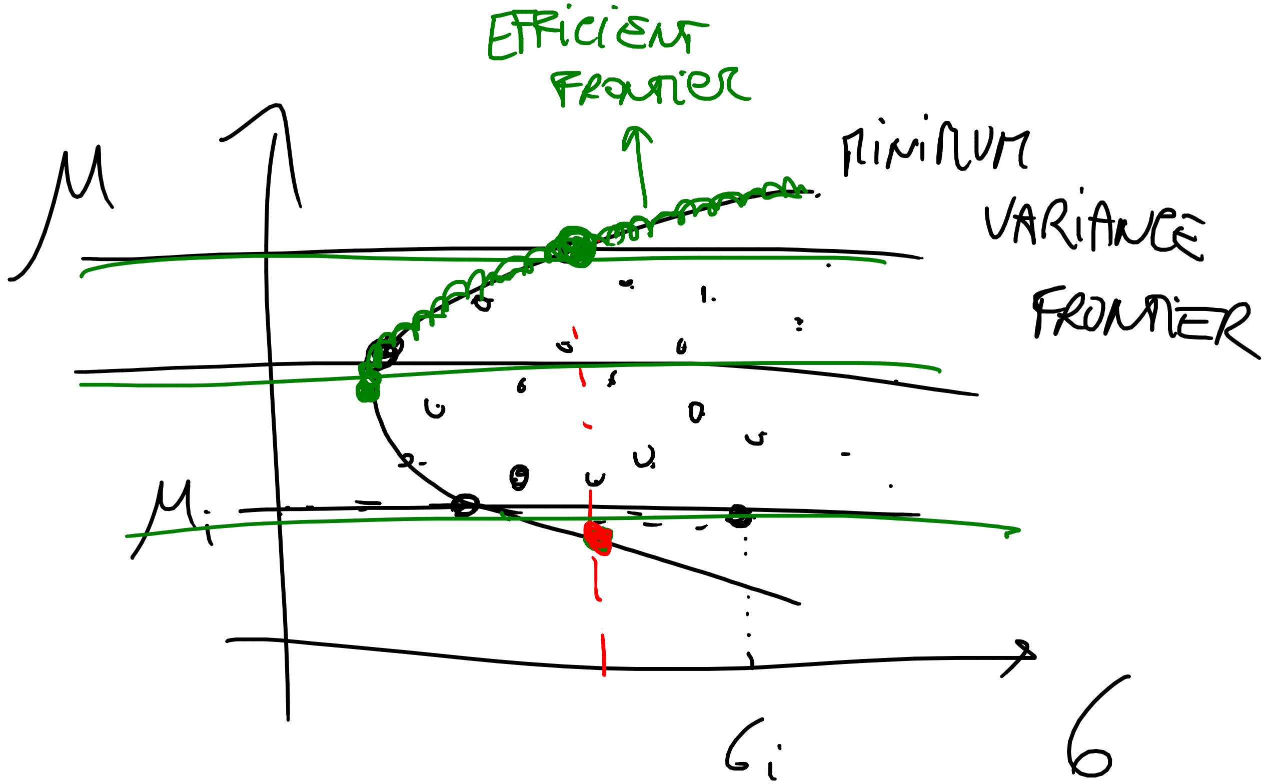
3) utility function (μ, σ^2)

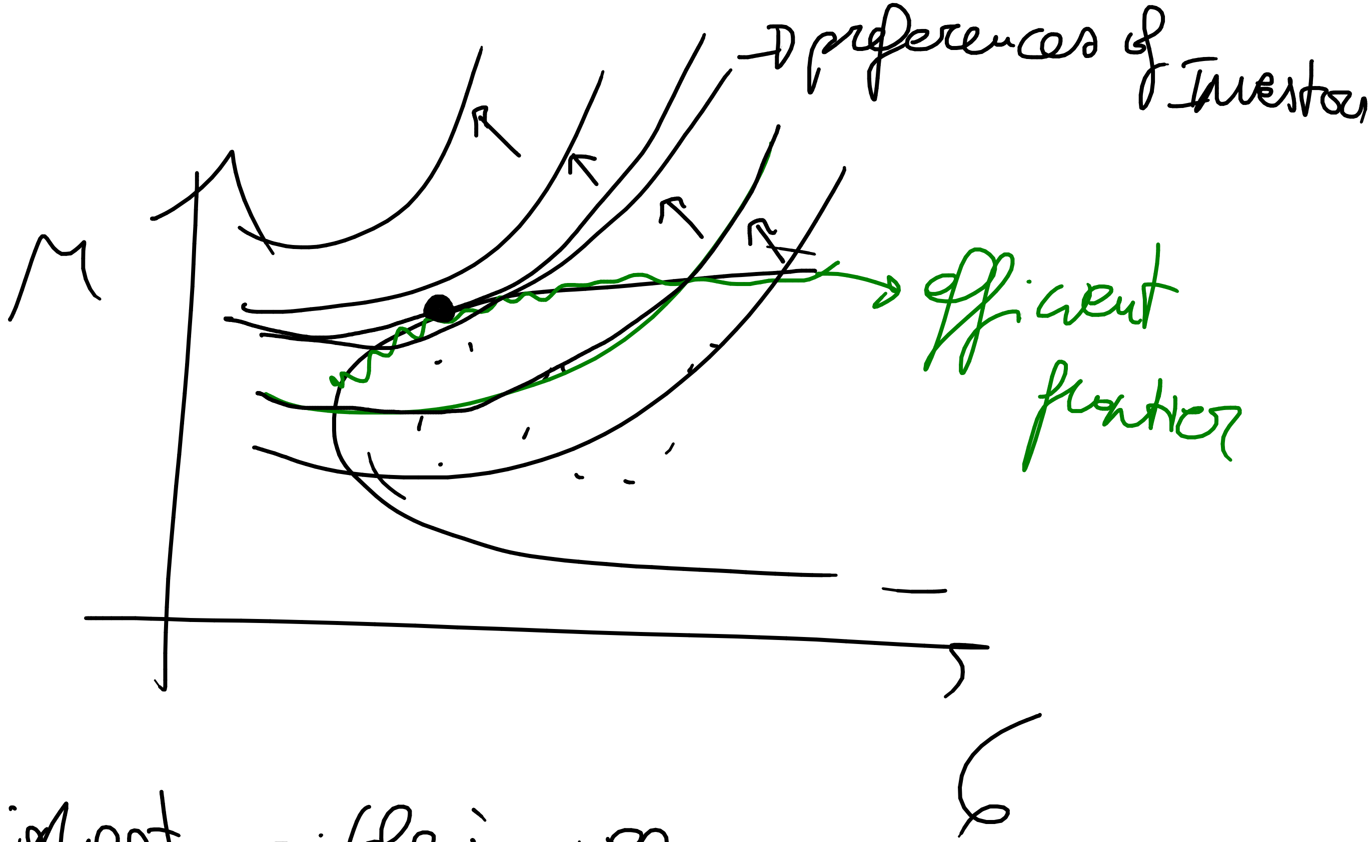






2 Assets

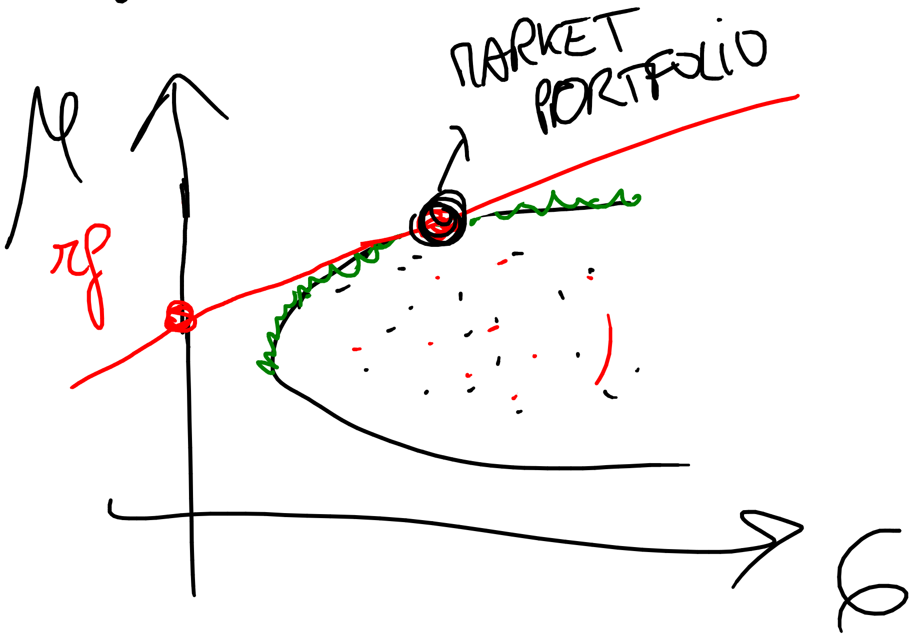

EFFICIENT FRONTIER
MINIMUM VARIANCE FRONTIER
μ
μ_i
σ_i
σ



highest possible indifference curve tangent to the efficient frontier

CAPM = CAPITAL ASSET PRICING MODEL

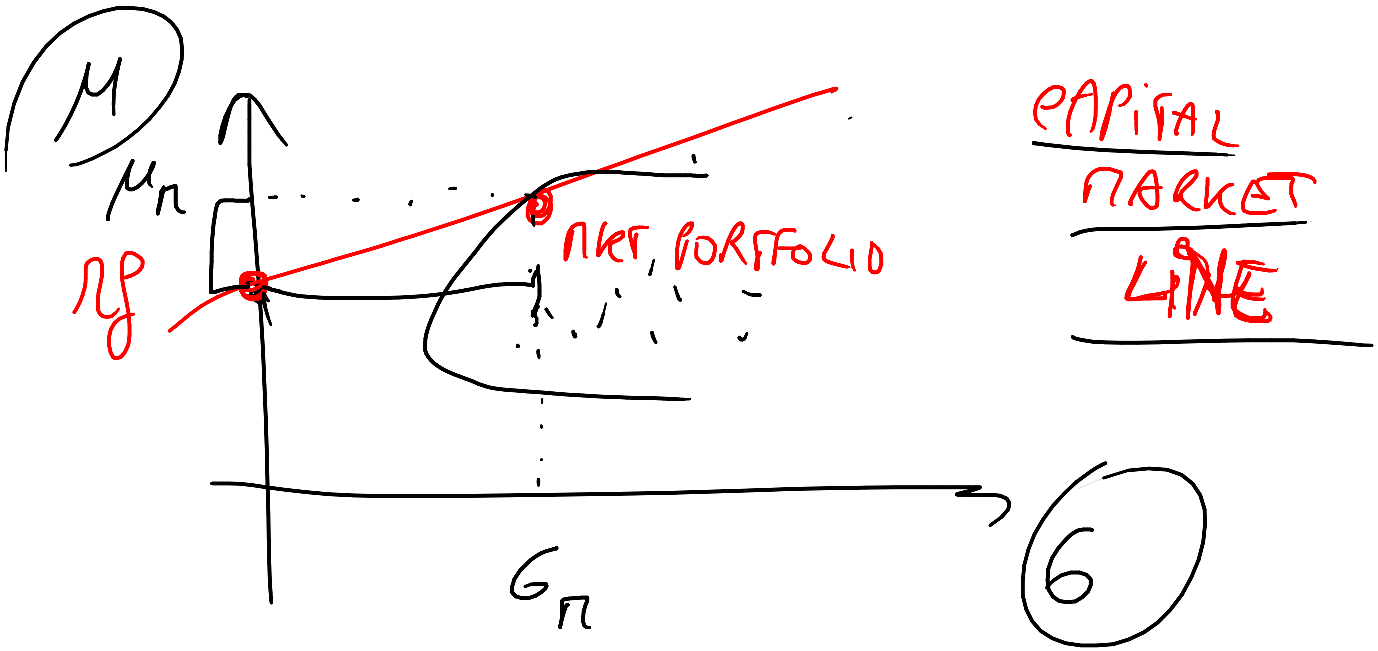
financial equilibrium :



1) μ, σ

2) demand = supply

3) $\exists$ risk free asset



$$E(R_i) = rf + \left[\frac{E(R_m) - rf}{\sigma_m}\right] \cdot \sigma_i$$

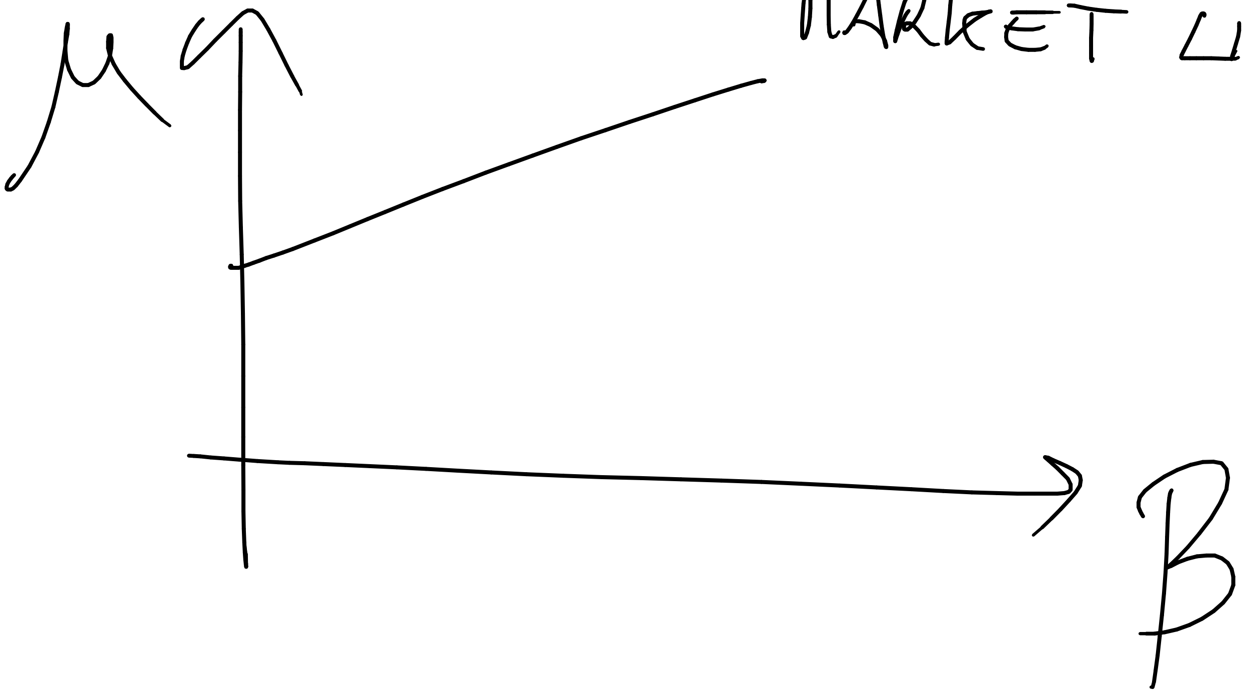
$$\parallel$$

$$\mu_i = rf + \left[\frac{\mu_m - rf}{\sigma_m}\right] \cdot \sigma_i$$

$$\underbrace{E(R_i) - r_f}_{\text{MARKET RISK PREMIUM}} = \underbrace{\boxed{\frac{E(R_m) - r_f}{\sigma_m}}}_{\text{PRICE OF RISK}} \cdot \underbrace{\boxed{\sigma_i}}_{\text{QUANTITY OF RISK}}$$

CML APPLIES ONLY TO EFFICIENT PORTFOLIOS

What can we say about other portfolios? → SECURITY MARKET LINE



$$E(R_i) = r_f + [E(R_m) - r_f] \cdot \beta_i$$

$$E(R_i) = r_f + [E(R_m) - r_f] \cdot \frac{\overbrace{cov(R_i, R_m)}}{\sigma_m^2}$$

$$cov(R_i, R_m) = \rho_{im} \cdot \sigma_i \cdot \sigma_m$$

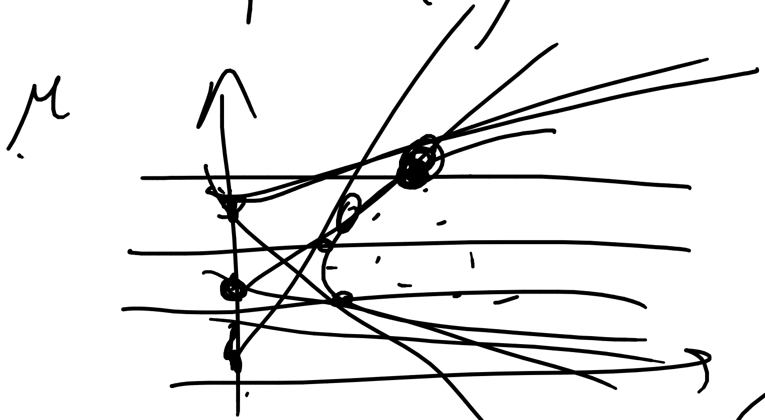
$$E(R_i) - r_f = [E(R_m) - r_f] \cdot \frac{\rho_{im} \cdot \sigma_i \cdot \cancel{\sigma_m}}{\cancel{\sigma_m^2}}$$

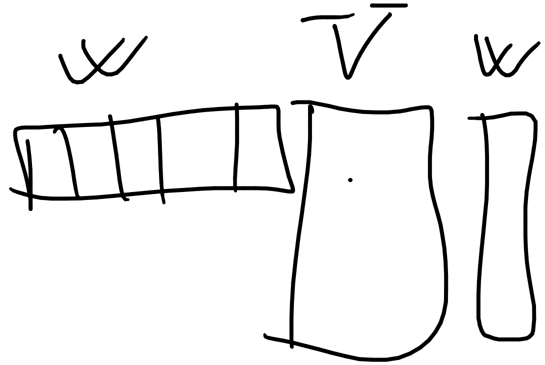
PRICE OF RISK

QUANTITY OF RISK only the PART ρ_{im} is PRICED

MINIMUM VARIANCE FRONTIER:

$\forall$ fixed μ I want min σ^2





σ

$$\begin{cases} \min \dfrac{1}{2} w^T \bar{V} w \\ \text{s.t.} \quad w^T \bar{R} + (1 - w^T) r_f = \mu \end{cases}$$

$$z = V^{-1}\,[E(R) - c]$$

NON NORMALIZED PROPORTION

inverse VAR-COV MATRIX

mean RETURNS

CONSTANT (risk free rate)

$$w_i = \frac{z_i}{\sum z_i}$$

USE TWO CONSTANTS

0

0,1

VAR-COV MATRIX

E(R)

E(R)-0

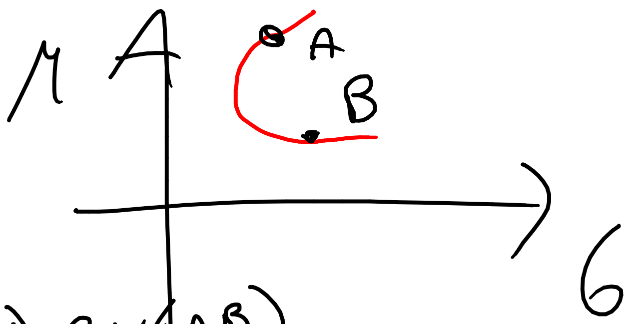
z_2

E(R)-0,1

$$z_1 = V^{-1} \cdot [E(R) - C]$$

= MMULT[Minverse(V); vector]

IF I HAVE 2 PORTFOLIOS WHICH ARE MINIMUM VARIANCE I CAN DERIVE ALL THE OTHERS ON THE MINIMUM VARIANCE FRONTIER BY A CONVEX COMBINATION OF THE TWO PORTFOLIOS



$$\mu_P = \alpha \cdot \mu_A + (1-\alpha) \cdot \mu_B$$

$$\sigma_P^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha) \cdot \mathrm{COV}(AB)$$

once I have $z_1 = V^{-1}[E(R) - 0,1]$

$$z_2 = V^{-1}[E(R)]$$

$$X_i = \frac{z_i^1}{\sum_1 z_i^1}$$

$$y_i = \frac{z_i^2}{\sum_1 z_i}$$